CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the May/June 2015 series

4037 ADDITIONAL MATHEMATICS

4037/22 Paper 2, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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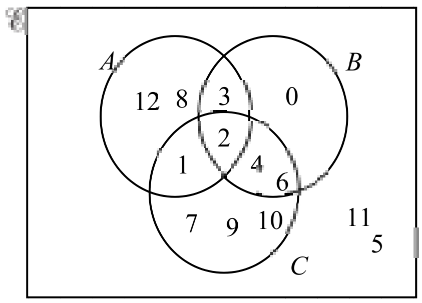
Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

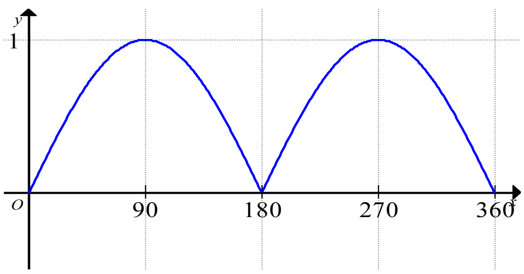
1 **(i)**		B3,2,1,0	2 correctly placed in Venn diagram; 1, 3, 4, 6 correctly placed; 12, 8, 0, 7, 9, 10 correctly placed; 11, 5 correctly placed
(ii)	3	B1ft	correct or correct ft *their* (i), provided non-zero
(iii)	{4, 6}	B1ft	correct or correct ft *their* (i), provided not the empty set
2 **(i)**	$[\mathbf{P} =]\begin{pmatrix} 60 & 70 & 58 \\ 50 & 52 & 34 \end{pmatrix}$ and $[\mathbf{Q} =]\ (120 \quad 300)$	B2	or $[\mathbf{P} =]\begin{pmatrix} 50 & 52 & 34 \\ 60 & 70 & 58 \end{pmatrix}$ and $[\mathbf{Q} =]\ (300 \quad 120)$ or B1 if one error may be written as an unevaluated product; B0 if choice of **P** and **Q** offered
(ii)	$(22\,200 \quad 24\,000 \quad 17\,160)$	B2	must have brackets and must not have commas; must be a 1 by 3 matrix; must be from correct product; working may be seen in **(i)** or B1 for any two elements correct
(iii)	The **total** (amount of revenue) **from all** (three) flights. oe	B1	do not accept, e.g. The total amount from **each** flight; must be a comment not just a figure; must not contain a contradiction

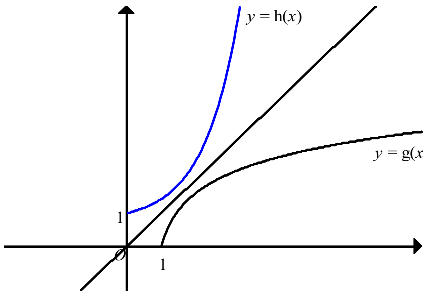
3 (i)	$\frac{(36+15\sqrt{5})}{(6+3\sqrt{5})}\times\frac{(6-3\sqrt{5})}{(6-3\sqrt{5})}$ oe	M1	or $\frac{(12+5\sqrt{5})}{(2+\sqrt{5})}\times\frac{(2-\sqrt{5})}{2-\sqrt{5}}$ oe
	$\frac{216+90\sqrt{5}-108\sqrt{5}-225}{-9}$	DM1	or $\frac{24+10\sqrt{5}-12\sqrt{5}-25}{-1}$ or $-(24+10\sqrt{5})-12\sqrt{5}-25$
	$1+2\sqrt{5}$ cao	A1	allow $a = 1$ and $b = 2$
	Alternative method: $36+15\sqrt{5}=(6a+15b)+(3a+6b)\sqrt{5}$	M1	
	$6a + 15b = 36$ $3a + 6b = 15$	DM1	
	$a = 1$ and $b = 2$	A1	or $1+2\sqrt{5}$
(ii)	$\left[AC^2=(6+3\sqrt{5})^2+their(1+2\sqrt{5})^2\right]$ $=36+36\sqrt{5}+45+their(1+4\sqrt{5}+20)$	M1	correct or correct ft expansions, using Pythagoras with $(6+3\sqrt{5})$ and *their BC*
	$102+40\sqrt{5}$ cao	A1	ignore attempts to square root after correct answer seen
4 (i)	$\cos(x)=\frac{2}{3}$ oe soi	M1	**Alternatively** $\sin(y)=\frac{2}{3}$ oe soi
	48.189…° or 131.810…° or 0.8410… rad or 2.3(00…) rad oe isw with reference axis indicated by comment, e.g. "to the bank" or "upstream", etc. or clearly marked on a diagram	A1	41.810…° or 0.7297 ... or 0.73(0) rad oe isw with reference axis indicated by comment, e.g. "to the perpendicular with the bank", etc. or clearly marked on a diagram If M0 then SC1 for an unsupported answer of 138.189…° or 2.4118… rad or 318.189...° or 5.5534... rad with reference axis indicated by comment, e.g. "on a bearing of" or "from North" or clearly marked on a diagram

(ii)	Speed $=\sqrt{9-4}\left(=\sqrt{5}\right)$ or $3\sin 48.2$ or $2\tan 48.2$ or $3\cos 41.8$ or $\dfrac{2}{\tan 41.8}$ or $\sqrt{2^2+3^2-2\times 2\times 3\cos 48.2}$ oe or 2.236(0…) rot to 4 or more figs or 2.24 [m/s] soi	B1	Or Distance $=\dfrac{80}{\sin 48.2}=107.(33\ldots)$ oe soi
	time $=\dfrac{80}{their\ \sqrt{5}}$ oe	M1	time $=\dfrac{their\ 107.33\ldots}{3}$
	35.66 to 35.8 (seconds) oe	A1	ignore subsequent rounding or attempted conversion to, e.g. minutes but A0 if answer spoiled by continuation of method if no working, so B0 M0, then allow B3 for an answer 35.66 to 35.8 oe
5	Substitution of either $4-x$ or $4-y$ into equation of curve and brackets expanded	M1	condone one sign error or slip in either equation of curve or expansion of brackets; condone omission of $=0$, BUT $4-x$ or $4-y$ must be correct
	$12x^2-52x+48$ [= 0] or $12y^2-44y+32$ [= 0] oe	A1	
	Solve their 3-term quadratic	M1	dep on a valid substitution attempt
	$x=\dfrac{4}{3}$ and 3 isw	A1	or $x=\dfrac{4}{3}$ $y=\dfrac{8}{3}$ not from wrong working
	$y=\dfrac{8}{3}$ and 1 isw	A1	or $x=3$ $y=1$ not from wrong working if no working, allow full marks for fully correct answer only.
6 (a)	$(x-2)\log 6=\log\left(\dfrac{1}{4}\right)$ oe or $\log_6\left(\dfrac{1}{4}\right)=x-2$ oe	M1	or $x\log 6=\log\left(\dfrac{36}{4}\right)$ oe or $x\log 6-\log 36=\log 1-\log 4$ oe
	1.23 or 1.226(29…) rot to 4 or more figures isw	A1	correct answer or 1.22 implies M1

(b)	**Method 1** $\log\left(\dfrac{8\times 2y^2\times 16y}{64y}\right)=\log 4^2$ oe	B3	or B2 if at most one error or omitted step or B1 if at most two errors or omitted steps
	$y = 2$	B1	not from wrong working
	Method 2 $\log 2 + 2\log y + 3\log 2 + 4\log 2 + \log y - 6\log 2 - \log y = 4\log 2$	B3,2,1,0	LHS terms $\log 2y^2 = \log 2 + 2\log y$; $\log 8 = 3\log 2$; $\log 16y = 4\log 2 + \log y$; $-\log 64y = -6\log 2 - \log y$; RHS term $2\log 4 = 4\log 2$
	$y = 2$	B1	not from wrong working
7	$\dfrac{n(n-1)(n-2)(n-3)(2^4)}{4\times 3\times 2\times 1}=10\dfrac{n(n-1)(2^2)}{2\times 1}$ or better	M3	condone omitting the factor of n and/or $n - 1$; must have dealt with factorials M2 if one slip/omission or M1 if two slips/omissions or B1 for $\dfrac{n(n-1)}{2}(2)^2[x^2]$ seen and B1 for $\dfrac{n(n-1)(n-2)(n-3)}{24}(2)^4[x^4]$ seen
	$n^2 - 5n - 24$ [= 0] oe	A1	equivalent must be 3-terms, e.g. $n^2 - 5n = 24$
	$(n + 3)(n - 8)$ [= 0]	M1	or any valid method of solution for their 3-term quadratic
	$n = 8$ only	A1	A0 if –3 also given as a final solution, i.e. not discarded If zero scored, allow SC1 for $n = 8$ unsupported or without correct method

8	**Method 1** (Separate areas subtracted)		
	$[x_B = x_C =]$ 7 soi	B1	
	$\left[\int(x^2 - 6x + 10)\,dx =\right] \frac{x^3}{3} - \frac{6x^2}{2} + 10x$	M2	or M1 for at least one term correct
	Correct or correct ft substitution of limits 0 and *their* 7 into *their* $\left[\frac{x^3}{3} - \frac{6x^2}{2} + 10x\right]$	DM1	dep on at least M1 being earned; evidence of substitution must be seen in *their* integral which must be at least two terms; condone omission of lower limit;
	$\frac{1}{2}(10+17)\times 7$ oe or $\int_0^7 (x+10)\,dx = \left[\frac{x^2}{2} + 10x\right]_0^7 = \frac{(7)^2}{2} + 10(7)$ oe	B2	or M1 for $\frac{1}{2}$(*their* 10 + *their* 17) × *their* 7 oe or B1 for $\int(x+10)dx = \frac{x^2}{2} + 10x$
	their $\left(\frac{189}{2} - \frac{112}{3}\right)$	M1	dep on a genuine attempt to integrate the equation of the curve; must be *their* area trapezium/under the line – *their* attempt at area under curve
	$\frac{343}{6}$ or $57\frac{1}{6}$ or 57.2 to 3 sf or 57.16(6...) rot to 4 figs isw	A1	from full and correct working with no omitted steps
	Method 2 (Subtracting and using integration once)		
	$[x_B = x_c =]$ 7 soi	B1	
	$\int(-x^2 + 7x)dx$	B1	condone omission of dx
	$\left[-\frac{x^3}{3} + \frac{7x^2}{2}\right]$ oe or $\left[\frac{x^3}{3} - \frac{7x^2}{2}\right]$ oe	M3	or M2 for $\int(px^2 + qx)dx = \frac{px^3}{3} + \frac{qx^2}{2}$ oe either with $p = \pm 1$ or $q = \pm 7$ or M1 for $\int(px^2 + qx)dx = \frac{px^3}{3} + \frac{qx^2}{2}$ with non-zero constants p and q, with $p \neq \pm 1$ and $q \neq \pm 7$
	Correct or correct ft substitution of limits 0 and *their* 7 into *their* $\left[-\frac{x^3}{3} + \frac{7x^2}{2}\right]$	M2	dep on a valid integration attempt; evidence of substitution must be seen; condone omission of lower limit;
	$\frac{343}{6}$ or $57\frac{1}{6}$ or 57.2 to 3 sf or 57.16(6...) rot to 4 figs isw	A1	from full and correct working with no omitted steps

9	(i)	$10 = 2m + 4$ soi	M1	or $[m =]\dfrac{10-4}{2-0}$ oe soi
		$m = 3$	A1	
	(ii)	1	B1	
	(iii)	$\dfrac{10-y_R}{2--1} = 1$ oe soi	M1	or $y = x + 8$ oe
		$(-1, 7)$ or $x = -1$ and $y = 7$	A1	if $y = 7$ only stated, provided that $x = -1$ is soi in working allow both marks if M0 then B1 for $y = 7$ only with no working
	(iv)	Use of $m_1\ m_2 = -1$ with *their* m from **(i)**	M1	may be implied by perpendicular gradient seen in equation
		$y - 10 = \left(their\ -\dfrac{1}{3}\right)(x-2)$	A1	or $\left(their\ -\dfrac{1}{3}\right)x + c$ and $10 = \left(their\ -\dfrac{1}{3}\right)2 + c$
		$3y + x = 32$ isw	A1	allow for correct equation with integer coefficients in any simplified form
	(v)	$\left(\dfrac{1}{2},\ their\dfrac{11}{2}\right)$ oe isw	B1,B1ft	ft *their* y_Q or M1 for $\left(\dfrac{2-1}{2}, \dfrac{10+1}{2}\right)$ seen
	(vi)	4.5 oe cao	B2	not from wrong working or M1 for any correct method with correct coordinates
10	(a)		B2,1,0	correct sinusoidal/reflected sinusoidal shape, all above x-axis with intent to have all maximum points of equal height; 2 maximum points of intended equal height only over 0 to 360; all max points clearly at $y = 1$; cusp at 180

(b)(i)	$[\text{hg}(x) =] \dfrac{e^{\ln(4x-3)}+3}{4}$	M1	**Alternative method** $y = \ln(4x-3)$ and change of subject to x oe,
	fully correct **and** completion to $[\text{hg}(x) =]\ x$	A1	fully correct and comment that $\text{h}(x) = \text{g}^{-1}(x)$ oe
(ii)		B2,1,0	correct shape; 1 marked on the y-axis or (0, 1) stated close by; curve with positive gradient in first quadrant only
(iii)	$x \geqslant 0$ or $[0, \infty)$	B1	not domain ≥ 0
(iv)	$y \geqslant 1$ or $[1, \infty)$	B1	or $\text{h}(x) \geqslant 1$, $\text{h} \geqslant 1$ etc.
11 (i)	$\dfrac{8-h}{8}$ or $8 : 8-h$ soi	M1	or $\dfrac{8}{8-h}$ or $8-h : 8$ soi
	$\dfrac{8-h}{8} \times 4$ oe	A1	or $4 \div \dfrac{8}{8-h}$ oe
	$h\left(\dfrac{8-h}{8} \times 4\right)^2$ oe	M1	h must be in the numerator of the expression for this mark;
	expand and simplify to $\dfrac{h^3}{4} - 4h^2 + 16h$ **AG**	A1	
(ii)	$\dfrac{3}{4}h^2 - 8h + 16$ oe	B1	
	their $\left(\dfrac{3}{4}h^2 - 8h + 16\right) = 0$ and attempt to solve	M1	must be a 3-term quadratic; must be an attempt at a derivative
	$\dfrac{8}{3}$ oe only	A2	or A1 for $h = \dfrac{8}{3}$ and 8 allow 2.67 or 2.66(6...) rot to 4 or more figs for $\dfrac{8}{3}$

12 (i)	$-120 + 104 + 22 - 6 = 0$ or correct unsimplified form, e.g. $15(-2)^3 + 26(-2)^2 - 11(-2) - 6 = 0$ or $15(-8) + 26(4) - 11(-2) - 6 = 0$	B1	or correct synthetic division $\begin{array}{r\|rrrr} -2 & 15 & 26 & -11 & -6 \\ & & -30 & 8 & 6 \\ \hline & 15 & -4 & -3 & 0 \end{array}$
(ii)	Substituting $x = 3$ into $15x^3 + 26x^2 - 11x - 6$	M1	or correct synthetic division $\begin{array}{r\|rrrr} 3 & 15 & 26 & -11 & -6 \\ & & 45 & 213 & 606 \\ \hline & 15 & 71 & 202 & 600 \end{array}$
	600	A1	correct answer implies M1; must be explicitly identified as answer if using synthetic/long division methods by e.g. circling
(iii)	$(x - 1)(15x^3 + 26x^2 - 11x - 6)$ soi	B1	by inspection or division; may be implied by e.g. $(ax + b)(15x^3 + 26x^2 - 11x - 6)$ and $a = 1$, $b = -1$ seen in later work comparing coefficients
	Multiply out $(x \pm 1)(15x^3 + 26x^2 - 11x - 6)$ and compare coefficients of x^3 or x to quartic	M1	or multiply out, e.g. $(ax + b)(15x^3 + 26x^2 - 11x - 6)$ and compare coefficients of x^3 or x to quartic
	$p = 11$ $q = 5$	A1 A1	correct p or q implies M1; correct p and q www implies B1 M1